KENDRIYA VIDYALAYA SANGATHAN
CHANDIGARH REGION

CAPSULE FOR LOW ACHIEVERS IN MATHEMATICS
CLASS : XII

1. MATRICES & DETERMINANTS:	**10 MARKS**
2. CONTINUTY:	**04 MARKS**
3. DIFFERENTIATION :	**04 MARKS**
4 .APPLICATION OF DERIVATIVES :	**10 MARKS**
(a) INCREASING & DECREASING FUNCTION	
(b) TANGENT & NORMAL	
(c) MAXIMA & MINIMA	
5. INDEFINITE INTEGRATION :	**04 MARKS**
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7. DIFFERENTIAL EQUATION :	**04 MARKS**
8. VECTOR ALGEBRA :	**04 MARKS**
9. THREE DIMENSIONAL GEOMETRY:	**10 MARKS**
10. LINEAR PROGRAMMING PROBLEM :	**06 MARKS**
11. PROBABILTY(BAYES THEOREM) :	**04 MARKS**
TOTAL :	**70 MARKS**

1. MATRICES & DETERMINANTS

* **Properties of Transpose** :

If A & B are matrices such that their sum & product are defined, then

(i). $\left(A^T\right)^T = A$ (ii). $(A+B)^T = A^T + B^T$ (iii). $\left(KA^T\right) = K.A^T$ where K is a scalar.

(iv). $(AB)^T = B^T A^T$ (v). $(ABC)^T = C^T B^T A^T$.

* **Symmetric Matrix** : A square matrix is said to be symmetric if $A^T = A$.

* **Skew symmetric Matrix** : A square matrix is said to be skew symmetric if $A^T = -A$.

*__Singular matrix__: A square matrix 'A' of order 'n' is said to be singular, if | A| = 0.

* **Non -Singular matrix** : A square matrix 'A' of order 'n' is said to be non-singular, if $|A| \neq 0$.

* Let A be a square matrix of order n × n, then $|kA| = k^n |A|$.

* **Area of a Triangle:** area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

* If A is a square matrix of order n, then $|adj(A)| = |A|^{n-1}$.

*__Inverse of a matrix__ :Inverse of a square matrix A exists, if A is non-singular or square matrix A is said to be invertible and $A^{-1} = \frac{1}{|A|}Adj.A$

* $AA^{-1} = A^{-1}A = I$
* $(A^{-1})^{-1} = A$
* $(AB)^{-1} = B^{-1}A^{-1}$
* $(A^T)^{-1} = (A^{-1})^T$

VERY SHORT ANSWER TYPE QUESTIONS (1 OR 2 Marks)

1. Construct a 2×2 matrix A = $[a_{ij}]$, whose elements are given by

(i) $a_{ij} = \frac{(i+j)^2}{2}$ (ii) $a_{ij} = \frac{1}{2}|i-3j|$ (iii) $a_{ij} = e^{2ix}\sin jx$

2. If A = diag [1, –2, 3] , B = diag [3, 4, –6] , and C = diag [0, 1, 2], find 4A – 2B + 3C.

3.Find x, y if $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

4.Compute the following products : (i) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

5.If A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, find f(A), where $f(x) = x^2 - 4x + 7$.

6.If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k so that $A^2 + 2I = kI$.

7. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then find the value of k.

ANSWERS

1(i) $\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$, (ii) $\begin{bmatrix} 1 & 5/2 \\ 1/2 & 2 \end{bmatrix}$, (iii) $\begin{bmatrix} e^{2x}\sin x & e^{2x}\sin 2x \\ e^{4x}\sin x & e^{4x}\sin 2x \end{bmatrix}$ **2**. diag [–5, –7, 21]

3. x = 2 and y = 9. **4**. (i) $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$ (ii) [82] **5**. 0 **6.** 1 **7.** 2

LONG-ANSWERTYPE QUESTIONS (4 MARKS)

1.If A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$. Hence find A^5.

2.Find the value of x if $[1 \quad x \quad 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

3.Let A = $\begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$, prove that : $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$.

4.Let $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Prove that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

5.Express $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Using properties of determinants, prove the following :(Q6 to Q 23)

6. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

7. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

8. $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix} = xy$

9. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

10. $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

11. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

12. $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

13. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$

14. $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

15. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

16. $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

17. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

18. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

19. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$

20. $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$ **21.** $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$

22. $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$ **23.** $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (ab+bc+ca)(a-b)(b-c)(c-a)$

24. If a, b, c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$,show that either a + b +c = 0 or a = b = c.

25. If a, b, c are positive and unequal, show that the value of the determinant $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

26. Show that A = $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x - 17 = 0$. Thus find A^{-1}.

27. If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

28. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = 0$. Hence, find A^{-1}.

29. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} P \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$.

30. Using elementary row transformations find A^{-1} : (i) $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, (ii) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

ANSWERS

1. $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$ **2.** x = –14 or –2 **5.** $\begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$

26. $\frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$ **27.** $\begin{matrix} x=8 \\ y=8 \end{matrix}$, $A^{-1} = \begin{bmatrix} 5/8 & -1/8 \\ -7/8 & 3/8 \end{bmatrix}$ **28.** $\begin{matrix} a=-4 \\ b=1 \end{matrix}$, $A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

29. $\begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$ **30.** (i) $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, (ii) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$,

LONG-ANSWERTYPE QUESTIONS (6 MARKS)

1. Solve the following system of equations by matrix method:

(i) x + 2y + z = 7, x + 3z = 11, 2x – 3y = 1. (ii) $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2,\ x,y,z \neq 0$

2. Find A^{-1}, where A = $\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$. Hence solve the equations x + 2y + 5z = 10, x –y – z = –2 and 2x + 3y –z = –11 .

3. Find A^{-1}, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Hence solve x + y + 2z = 0, x + 2y – z = 9 and x – 3y + 3z = –14

4. Find the matrix P satisfying the matrix equation : $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

5. Given that $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB. Use this product to solve the following system of equations: x – y = 3 ; 2 x + 3 y + 4 z = 17 ; y + 2 z = 7.

6. Using elementary transformations find A^{-1}:(i) $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

ANSWERS

1.(i) $A^{-1} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$, $x = 2,\ y = 1,\ z = 3.$ **(ii)** $A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$, $x = 2,\ y = 3,\ z = 5.$

2. $A^{-1} = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$, $x = -1,\ y = -2,\ z = 3.$ **3.** $A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$, $x = 1,\ y = 3,\ z = -2$

4. $P = A^{-1}QB^{-1} \Rightarrow P = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 1 & -8 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

5. x = 2, y = – 1 and z = 4

6. (i) $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ (iii) $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ (v) $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

2. CONTINUTY

* A function f is said to be continuous at x = a if

Left hand limit = Right hand limit = value of the function at x = a

i.e. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$ i.e. $\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+h) = f(a)$.

* A function is said to be differentiable at x = aif $Lf'(a) = Rf'(a)$ i. $\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

LONG-ANSWERTYPE QUESTIONS (4 MARKS)

1. If $f(x) = \begin{cases} 3ax+b, & \text{if } x > 1 \\ 11 & \text{if } x = 1 \\ 5ax-2b, & \text{if } x < 1 \end{cases}$, continuous at x = 1, find the values of a and b.

2. Determine a, b, c so that f(x) = $\begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}, & x<0 \\ c, & x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & x>0 \end{cases}$ is continuous at x = 0.

3. If f(x) = $\begin{cases} \dfrac{k\cos x}{\pi-2x}, & x \ne \dfrac{\pi}{2} \\ 3, & x=\dfrac{\pi}{2} \end{cases}$, is continuous at x = $\dfrac{\pi}{2}$, find k.

4. f(x) = $\begin{cases} 2x+1, & x<2 \\ k, & x=2 \\ 3x-1, & x>2 \end{cases}$ is continuous at x = 2, find k.

5. Find the values of a and b such that the function defined byf(x) = $\begin{cases} 1, & \text{if } x \le 3 \\ ax+b, & \text{if } 3<x<5 \\ 7, & \text{if } x \ge 5 \end{cases}$ is a continuous function.

6. If f(x) = $\begin{cases} \dfrac{x-4}{|x-4|}+a, & \text{if } x<4 \\ a+b & \text{if } x=4 \\ \dfrac{x-4}{|x-4|}+b & \text{if } x>4 \end{cases}$ is continuous at x = 4, find a, b.

7. The function f(x) is defined as follows : f(x) = $\begin{cases} x^2+ax+b, & \text{if } 0 \le x<2 \\ 3x+2, & \text{if } 2 \le x \le 4 \\ 2ax+5b, & \text{if } 4<x \le 8 \end{cases}$.

If f(x) is continues on [0 , 8], find the value of a & b.

8. If f(x) = $\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2} \\ a & \text{if } x=\dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & \text{if } x>\dfrac{\pi}{2} \end{cases}$ is continuous at x = $\dfrac{\pi}{2}$, find a, b.

9. Discuss the continuity of f(x) = $|x-1|+|x-2|$ at x = 1 & x = 2.

ANSWERS

1. $a=3, b=2$ **2.** $a=-\dfrac{3}{2}, c=\dfrac{1}{2}$, b is any non - zero real number . **3.** $k=6$

4.k = 5 **5.**a = 3, b = – 8 **6.** a = 1, b = – 1 **7.** $a=3, b=-2$

8. $a=\dfrac{1}{2}, b=4$ **9.**Continuous at x = 1 & 2.

3. DIFFERENTIATION

(i) $\dfrac{d}{dx}(x^n) = n\,x^{n-1}$. , $\dfrac{d}{dx}\left(\dfrac{1}{x^n}\right)=-\dfrac{n}{x^{n+1}}$ (ii) $\dfrac{d}{dx}(x) = 1$

(iii) $\dfrac{d}{dx}(c) = 0, \forall\, c \in R$ (iv) $\dfrac{d}{dx}(a^x) = a^x \log a$, a > 0, a ≠ 1.

(v) $\dfrac{d}{dx}(e^x) = e^x$. (vi) $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x \log a}$,a> 0, a ≠ 1

(vii) $\frac{d}{dx}(\log x) = \frac{1}{x}$, $x > 0$

(viii) $\frac{d}{dx}(\log_a|x|) = \frac{1}{x \log a}$, $a > 0, a \neq 1, x \neq 0$

(ix) $\frac{d}{dx}(\log|x|) = \frac{1}{x}$, $x \neq 0$

(x) $\frac{d}{dx}(\sin x) = \cos x$

(xi) $\frac{d}{dx}(\cos x) = -\sin x$

(xii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(xiii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(xiv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(xv) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(xvi) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$.

(xvii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$.

(xviii) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$,

(xix) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(xx) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$.

(xxi) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$.

(xxii) $\frac{d}{dx}(|x|) = \frac{x}{|x|}$, $x \neq 0$

(xxiii) $\frac{d}{dx}(ku) = k\frac{du}{dx}$

(xxiv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(xxv) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

(xxvi) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

LONG-ANSWERTYPE QUESTIONS (4 MARKS)

1. If $x^p.y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

2. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ w.r.t. $\cos^{-1} x^2$.

3. Differentiate w. r. t. x : $x^{\sin x} + (\sin x)^{\cos x}$

4. If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

5. If $(\cos x)^y = (\sin y)^x$ find $\frac{dy}{dx}$.

6. If $x = a(\cos\theta + \theta \sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$ find $\frac{d^2y}{dx^2}$ at $\theta = \pi/4$.

7. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

8. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

9. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$, $y = a\sin\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

10. If $y = (\sin^{-1} x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$.

11. Find $\frac{d}{dx}(a^x + e^x + x^x + x^a + a^a)$

12. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ w.r.t. $\cos^{-1} x^2$.

13. If $y=\left(x+\sqrt{x^2+a^2}\right)^n$, prove that $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$

14. If = $(\tan^{-1}x)^2$, prove that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx}-2=0$

15. Find $\frac{dy}{dx}$ if y = $\cos^{-1}\left[\frac{3x+4\sqrt{1-x^2}}{5}\right]$.

16. If $y=\sqrt{x^2+1}-\log\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.

17. If $y=e^{\tan x}$, prove that $\cos^2 x\frac{d^2y}{dx^2}-(1+\sin 2x)\frac{dy}{dx}=0.$

ANSWERS

2. $-\frac{1}{2}$

3. $x^{\sin x}\left[\frac{\sin x}{x}+\log x.\cos x\right]+(\sin x)^{\cos x}[\cos x.\cot x-\sin x\log\sin x]$

4. $(\sin x)^x[\log\sin x+x\cot x]+\frac{1}{2\sqrt{x-x^2}}$.

5. $\frac{\log\sin y+y\tan x}{\log\cos x-x\cot y}$

6. $\frac{d^2y}{dx^2}=-\frac{\operatorname{cosec}^2\theta}{a\cos\theta},\left[\frac{d^2y}{dx^2}\right]_{\pi/4}=\frac{8\sqrt{2}}{\pi a}$

9. $\frac{d^2y}{dx^2}=\frac{\sin\theta}{a\cos^4\theta},\left[\frac{d^2y}{dx^2}\right]_{\theta=\pi/3}=\frac{8\sqrt{3}}{a}$

11. $a^x\log a+e^x+x^x(1+\log x)+ax^{a-1}$

15. $\frac{-1}{\sqrt{1-x^2}}$

16. $\frac{\sqrt{x^2+1}}{x}$

4 .APPLICATION OF DERIVATIVES

(a) INCREASING & DECREASING FUNCTION

** (i) f is strictly increasing in (a, b) if $f'(x) > 0$ for each $x \in (a, b)$

(ii) f is strictly decreasing in (a, b) if $f'(x) < 0$ for each $x \in (a, b)$

LONG-ANSWERTYPE QUESTIONS (4 MARKS)

1. Find the intervals in which the following functions are strictly increasing or decreasing:

(i) $f(x) = 4x^3 - 6x^2 - 72x + 30$

(ii) $f(x) = -2x^3 - 9x^2 - 12x + 1$

(iii) $f(x) = (x + 1)^3 (x - 3)^3$

(iv) $f(x) = \sin x + \cos x$, $0\le x\le 2\pi$

(v) $f(x) = \sin 3x$, $x \in \left[0,\frac{\pi}{2}\right]$

(vi) $f(x) = \frac{4\sin x-2x-x\cos x}{2+\cos x}$

(vii) $f(x) = x^3+\frac{1}{x^3}$, $x\neq 0$

2.Show that y = $\log(1+x)-\frac{2x}{2+x}$, $x>-1$, is an increasing function of xthroughout its domain.

3. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function in $\left[0,\frac{\pi}{2}\right]$.

ANSWERS

1.(i) ↑ in $(-\infty,-2)\cup(3,\infty)$ and ↓in (–2 , 3) (ii) ↑in (– 2 , – 1) and ↓in $(-\infty,-2)\cup(-1,\infty)$

(iii)↑in (1 , 3) ∪ (3 , ∞) and ↓ in (– ∞ ,– 1) ∪ (–1, 1)

(iv)↑in $\left[0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right]$ and ↓ in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$ (v) ↑in $\left[0,\frac{\pi}{6}\right]$ and ↓ in $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$

(vi) ↑ in $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right)$ and ↓ in $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ (vii) ↑ in $(-\infty,-1)\cup(1,\infty)$ and ↓ in (– 1, 1)

(b) TANGENT & NORMAL

** The equation of the tangent at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = f'(x_0)(x - x_0)$.

** The equation of the normal at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$.

LONG-ANSWERTYPE QUESTIONS (4 MARKS)

1. Find points on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

2. Find the equations of the tangent and normal to the curvey = $x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5).

3.Find the equation of the tangent line to the curve y = $x^2 - 2x + 7$ which is

(a) parallel to the line 2x – y + 9 = 0 (b) perpendicular to the line 5y – 15x = 13.

4. For the curve y = $4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

5. Find the equation of the normals to the curve y = $x^3 + 2x + 6$ which are parallel to the line x + 14y + 4 = 0.

6. Prove that the curves x = y^2 and xy = k cut at right angles if $8k^2 = 1$.

7. Show that the normal at any point θ to the curve x = a cosθ + aθsinθ, y = a sinθ – aθcosθ is at a constant distance from the origin.

ANSWERS

1.(i) (0, 5) and (0, – 5) (ii) (2, 0) and (–2, 0). 2.Tangent : 10x + y = 5, Normal : x – 10y + 50 = 0.

3. (a) y – 2x – 3 = 0 , (b) 36y + 12x – 227 = 0. 4. (0 , 0) , (1 , 2), (– 1, – 2)

5. x + 14y – 254 = 0, x + 14y + 86 = 0.

(c) MAXIMA & MINIMA

****** If $c_1, c_2, \dots, c_n$ are the critical points lying in [a , b],
then absolute maximum value of f = max{f(a), f(c_1), f(c_2), … , f(c_n), f(b)}
and absolute minimum value of f =min{f(a), f(c_1), f(c_2), … , f(c_n), f(b)}.

** Second derivative test

(i) a function has a local maxima at x = a if f ′(x) 0 and f ″ (a) <0

(ii) a function has a local minima at x = a if f ′(x) = 0 andf ″(a) > 0.

LONG-ANSWERTYPE QUESTIONS (4 or 6MARKS)

1.A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

2. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

3.A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

4.Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

5.Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

6.Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

7.Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

8. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

9. A window is in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

10. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height hand semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

11. The sum of the perimeter of a circle and square is k, where kis some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

12. If the length of three sides of a trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum.

13. Show that the height of a cylinder, which is open at the top, having a given surface and greatest volume, is equal to the radius of the base.

14. If the sum of hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is π/3.

15. Find the point on the curve $y^2 = 4x$, which is nearest to the point (2, –8).

ANSWERS

1. 3 cm

3. $\frac{112}{\pi+4}$ cm , $\frac{28\pi}{\pi+4}$ cm.

8. $l = \frac{20}{\pi+4}$ m , $b = \frac{10}{\pi+4}$ m .

9. $\frac{12}{6-\sqrt{3}}, \frac{18-6\sqrt{3}}{6-\sqrt{3}}$

12. $\frac{75\sqrt{3}}{2}\text{cm}^2$

15. (4 , –8).

5. INDEFINITE INTEGRATION

* $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

* $\int 1.dx = x + c$

* $\int \frac{1}{x^n} dx = -\frac{1}{(n-1)x^{n-1}} + c$

* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + c$

* $\int \frac{1}{x} dx + c$

* $\int e^x dx = e^x + c$

* $\int a^x dx = \frac{a^x}{\log a} + c$

* $\int \sin x dx = -\cos x + c$

* $\int \cos x dx = \sin x + c$

* $\int \sec^2 x\, dx = \tan x + c$

* $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

* $\int \sec x.\tan x\, dx = \sec x + c$

* $\int \operatorname{cosec} x.\cot x\, dx = -\operatorname{cosec} x + c$

* $\int \tan x\, dx = -\log|\cos x| + c = \log|\sec x| + c$

* $\int \cot x\, dx = \log|\sin x| + C$

* $\int \sec x\, dx = \log|\sec x + \tan x| + C$

* $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + C$

* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$, if x > a

* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$, if $x > a$

* $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

* $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c$

* $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log | x + \sqrt{x^2 + a^2} | + C$

* $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log | x + \sqrt{x^2 - a^2} | + C$

* $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + C$

* $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$

* $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

LONG-ANSWERTYPE QUESTIONS (4 MARKS)

****TYPE-I** $\int \frac{px+q}{ax^2+bx+c} dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$, $\int (px+q)\sqrt{ax^2+bx+c}\, dx$

1. $\int \frac{(2x-3)dx}{x^2-3x-18}$
2. $\int \frac{2\sin 2\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi} d\phi$
3. $\int \frac{x^2.dx}{x^2+6x+12}$
4. $\int \frac{(3x+1).dx}{\sqrt{5-2x-x^2}}$
5. $\int \frac{(2x+1).dx}{\sqrt{x^2+4x+3}}$
6. $\int (2x+3)\sqrt{x^2+4x+3}.dx$
7. $\int \frac{dx}{\sqrt{(x-a)(x-b)}}$

**** <u>TYPE-II</u> Integration of Trigonometric Functions**

* $2\sin A.\cos B = \sin(A+B) + \sin(A-B)$
* $2\cos A.\cos B = \sin(A+B) - \sin(A-B)$
* $2\cos A.\cos B = \cos(A+B) + \cos(A-B)$
* $2\sin A.\sin B = \cos(A-B) - \cos(A+B)$
* $\cos^2 A = \frac{1+\cos 2A}{2}$, $\sin^2 A = \frac{1-\cos 2A}{2}$
* $\sin 3x = 3\sin x - 4\sin^3 x$, $\cos 3x = 4\cos^3 x - 3\cos x$

1. $\int \sin^4 x.dx$
2. $\int \cos^4 x.dx$
3. $\int \cos x.\cos 2x.\cos 3x.dx$
4. $\int \sin^3 x.\cos^3 x.dx$
5. $\int \sin^5 x.dx$
6. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x.\cos^2 x} dx$

**** <u>TYPE-III</u>** $\int \frac{dx}{a\sin^2 x + b\cos^2 x}$, $\int \frac{dx}{a + b\sin^2 x}$ $\int \frac{dx}{a + b\cos^2 x}$, $\int \frac{dx}{(a\sin x + b\cos x)^2}$, $\int \frac{dx}{a + b\sin^2 x + c\cos^2 x}$

(i) $\int \frac{dx}{a\sin^2 x + b\cos^2 x}$ (ii) $\int \frac{dx}{\cos x(\sin x + 2\cos x)}$

**** <u>TYPE-IV</u>** $\int \frac{dx}{a\sin x + b\cos x}$, $\int \frac{dx}{a\sin x + b\cos x + c}$, $\int \frac{dx}{a + b\cos x}$, $\int \frac{dx}{a + b\sin x}$

1. $\int \frac{dx}{3 + 2\sin x + \cos x}$
2. $\int \frac{dx}{5 + 4\sin x}$
3. $\int \frac{dx}{\sin x + \sqrt{3}\cos x}$
4. $\int \frac{dx}{\sin x + \sqrt{3}\cos x}$

**** <u>TYPE-V</u>** $\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x} dx$, $\int \frac{dx}{a \pm b\sin x}$, $\int \frac{dx}{a \pm b\cos x}$

1. $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$
2. $\int \frac{dx}{1 - \tan x}$
3. $\int \frac{dx}{1 + \cot x}$

**** <u>TYPE-VI</u> Integration by parts :**

1. $\int x^2 \tan^{-1} x\, dx$ 2. $\int (\sin^{-1} x)^2 dx$ **3.** $\int (\log x)^2 dx$ **4.** $\int \sec^3 x\, dx$

** **TYPE-VII** $\int e^x [f(x) + f(x)']dx$

1. $\int \frac{xe^x}{(x+1)^2} dx$ **2.** $\int \left(\frac{2+\sin x}{1+\cos 2x}\right) e^x dx$ **3.** $\int \left(\frac{1-\sin x}{1-\cos x}\right) e^x dx$ **4.** $\int \frac{x^2+1}{(x+1)^2} e^x dx$

5. $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$ **6.** $\int \frac{x-1}{(x+1)^3} e^x dx$ **7.** $\int \frac{2-x}{(1-x)^2} dx$

** **TYPE-VIII** $\int e^{ax} \sin bx\, dx$, $\int e^{ax} \cos bx\, dx$

1. $\int e^{2x} \sin 3x\, dx$ 2. $\int e^x \sin^2 x\, dx$

** **TYPE-IX** Integration of Rational Functions using **PARTIAL FUNCTION**

1. $\int \frac{x^2}{(x-1)(x-2)(x-3)} dx$ 2. $\int \frac{3x-2}{(x+1)^2(x+3)} dx$ 3. $\int \frac{dx}{(x+1)^2(x^2+1)} dx$ 4. $\int \frac{1}{1+x^3} dx$

5. $\int \frac{x^2+2x+8}{(x-1)(x-2)} dx$ 6. $\int \frac{x^2+x+1}{(x^2+1)(x+2)} dx$ 7. $\int \frac{dx}{\sin x + \sin 2x}$ 8. $\int \frac{\tan x + \tan^3 x}{1+\tan^3 x} dx$

9. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$ 10. $\int \frac{1}{x(x^4-1)} dx$

** **TYPE-X**

1. $\int \frac{dx}{\sin(x-a).\sin(x-b)}$ 2. $\int \frac{dx}{\cos(x-a).\cos(x-b)}$ 3. $\int \frac{dx}{\sin(x+a).\cos(x+b)}$ 4. $\int \frac{\cos(x+a)}{\sin(x+b)}.dx$

ANSWERS

TYPE 1

1. $\left[\log\left|x^2+3x-18\right| - \frac{2}{3}\log\left|\frac{x-3}{x+6}\right| + c\right]$ 2. $\left[2\log\left|\sin^2\phi - 4\sin\phi + 5\right| + 7\tan^{-1}(\sin\phi - 2) + c\right]$

3. $\left[x - 3\log\left|x^2+6x+12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c\right]$ 4. $\left[-3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c\right]$

5. $\left[2\sqrt{x^2+4x+3} - 3\log\left[x + 2\sqrt{x^2+4x+3}\right] + c\right]$

6. $\left[\frac{2}{3}\left(x^2+4x+3\right) - \frac{1}{2}\left\{(x+2)\sqrt{x^2+4x+3} - \log\left|(x+2) + \sqrt{x^2+4x+3}\right|\right\} + c\right]$

7. $\left[2\log\left|\sqrt{x-a} + \sqrt{x-b}\right| + c\right]$

TYPE-II

1. $\left[\frac{1}{8}\left(3x - 2\sin 2x + \frac{\sin 4x}{4}\right) + c\right]$ 2. $\left[\frac{1}{8}\left(3x + 2\sin 2x + \frac{\sin 4x}{4}\right) + c\right]$ 3. $\left[\frac{x}{4} + \frac{\sin 6x}{24} + \frac{\sin 4x}{16} + \frac{\sin 2x}{8} + c\right]$

4. $\left[\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + c\right]$ 5. $\left[\sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + c\right]$ 6. $\left[-\frac{1}{2}\sin 2x + c\right]$

TYPE-III

1. $\left[\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right) + c\right]$ 2. $\left[\log(\tan x + 2) + c\right]$

TYPE-IV

1. $\left[\tan^{-1}\left(1 + \tan\frac{x}{2}\right) + c\right]$ 2. $\left[\frac{2}{3}\tan^{-1}\left(\frac{\tan x/2}{3}\right) + c\right]$ 3. $\left[\frac{1}{2}\log\left|\frac{1+\sqrt{3}\tan x/2}{3-\sqrt{3}\tan x/2}\right| + c\right]$

4. $\left[\frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right|+c\right]$

TYPE-V

1. $\left[\frac{18}{15}x+\frac{1}{25}\log|3\sin x+4\cos x|+c\right]$ 2. $\left[\frac{x}{2}-\frac{1}{2}\log|\sin x-\cos x|\right]$ 3. $\left[\frac{x}{2}-\frac{1}{2}\log|\sin x-\cos x|\right]$

TYPE-VI

1. $\frac{x^3}{3}\tan^{-1}x-\frac{1}{6}x^2+\frac{1}{6}\log\left|x^2+1\right|+C$ 2. $x(\sin^{-1}x)^2-2\left[-\sin^{-1}x.\sqrt{1-x^2}+x\right]+C$

3. $x(\log x)^2-2[x\log x-x]+C$ 4. $\frac{1}{2}\sec x\tan x+\frac{1}{2}\log|\sec x+\tan x|+C$

** **TYPE-VII**

1. $\frac{1}{x+1}e^x+C$ 2. $e^x\tan x+C$ 3. $-e^x\cot\frac{x}{2}+c$

4. $e^x-2\frac{e^x}{x+1}+C$ **5.** $x\log(\log x)-\frac{x}{\log x}+C$ 6. $\frac{e^x}{(x+1)^2}+C$

7. $\frac{e^x}{1-x}+C$

** **TYPE-VIII**

1. $\frac{e^{2x}}{13}(2\sin 3x-3\cos 3x)+C$ 2. $\frac{1}{2}e^x-\frac{e^x}{10}(\cos 2x+2\sin 2x)+C$

TYPE-IX

1. $\frac{1}{2}\log|x-1|-4\log|x-2|+\frac{9}{2}\log|x-3|+C$ 2. $\frac{11}{4}\log\left|\frac{x+1}{x+3}\right|+\frac{5}{2}\left(\frac{1}{x+1}\right)+C$

3. $\frac{1}{2}\log|x+1|-\frac{1}{2(x+1)}-\frac{1}{4}\log\left|x^2+1\right|+C$ 4. $\frac{1}{3}\log|1+x|-\frac{1}{6}\log\left|1-x+x^2\right|+\frac{1}{2}.\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+C$

5. $x-11\log|x-1|+16\log|x-2|+C$ 6. $\frac{3}{5}\log|x+2|+\frac{1}{5}\log(x^2+1)+\frac{1}{5}\tan^{-1}x+C$

7. $\frac{1}{6}\log|\cos x-1|+\frac{1}{2}\log|\cos x+1|-\frac{2}{3}\log|1+2\cos x|+C$

8. $-\frac{1}{3}\log|1+\tan x|+\frac{1}{6}\log\left|\tan^2x-\tan x+1\right|+\frac{1}{\sqrt{3}}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$

9. $x+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-3\tan^{-1}\left(\frac{x}{2}\right)+C$ 10. $\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+C$

TYPE-X

1. $\frac{1}{\sin(b-a)}\log\frac{\sin|x-b|}{\sin|x-a|}+C$ 2. $\frac{1}{\sin(b-a)}\log\frac{\cos|x-b|}{\cos|x-a|}+C$

3. $\frac{1}{\cos(b-a)}\log\left|\frac{\sin(x+a)}{\cos(x+b)}\right|+C$ 4. $\cos(a-b)\log\sin|x+b|-(x+b)\sin(b-a)+C$

6. DEFINITE INTEGRATION

* **General Properties of Definite Integrals.**

* $\int_a^b f(x)\,dx=\int_a^b f(t)\,dx$ * $\int_a^b f(x)\,dx=-\int_a^b f(x)\,dx$

* $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ * $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

* $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

* $\int_{-a}^a f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$

* $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x). \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$.

LONG-ANSWERTYPE QUESTIONS (4 or 6MARKS)

Evaluate the following integrals

1. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

2. $\int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x+\cos^3 x}dx$

3. $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$

4. $\int_0^{\pi} \frac{x}{1+\sin x}dx$

5. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$ OR $\int_0^{\pi} \frac{x\sin x}{1+\sin x}dx$

6. $\int_0^{\pi} \frac{x\tan x}{\sec x+\cos x}dx$ OR $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

7. $\int_0^{\pi/2} \log\sin x\,dx$

8. $\int_0^{\pi} \frac{x\,dx}{4-\cos^2 x}$

9. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

10. $\int_0^{\pi/4} \log(1+\tan x)dx$

11. $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx$

12. $\int_0^{\pi/2} \frac{x\sin x.\cos x}{\sin^4 x+\cos^4 x}dx$

13. $\int_0^{\pi} \frac{x\tan x}{\sec x.\operatorname{cosec} x}dx$

14. $\int_0^{\pi/2} \frac{\cos^2 x}{\sin x+\cos x}dx$

15. $\int_0^{\pi} \frac{x}{1+\cos\alpha.\sin x}dx$

16. $\int_0^{\pi} \log(1+\cos x)dx$

17. $\int_{-1}^{2} |x^3 - x|dx$

18. $\int_2^5 (|x-2|+|x-3|+|x-4|)\,dx$

19. $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

20. $\int_0^{\pi/2} \sin 2x.\tan^{-1}(\sin x)dx$

21 $\int_{-1}^{3/2} |x\sin \pi x|\,dx$

22. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|]dx$

23 $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$

24. $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x+\cos^4 x}$

25. $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$

26. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x+4\cos^2 x}dx$

ANSWERS

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{4}$ 3. 1 4. π 5. $\pi\left(\frac{\pi}{2}-1\right)$ 6. $\frac{\pi^2}{4}$

7. $-\frac{\pi}{2}\log 2$ 8. $\frac{\pi^2}{4\sqrt{3}}$ 9. $\frac{\pi}{12}$ 10. $\frac{\pi}{8}\log 2$ 11. $\frac{\pi}{2}\log\frac{1}{2}$ 12. $\frac{\pi^2}{16}$

13. $\frac{\pi^2}{4}$ 14. $\frac{\log(\sqrt{2}+1)}{\sqrt{2}}$ 15. $\frac{\pi\alpha}{\sin\alpha}$ 16. $-\pi\log 2$ 17. $\frac{11}{4}$ 18. $\frac{19}{2}$

19. $\frac{1}{40}\log 9$ 20. $\frac{\pi}{2}-1$ 21. $\frac{3\pi+1}{\pi^2}$ 22. 0 23. $\frac{\pi}{4}$ 24. $\frac{\pi}{2}$

25. $\frac{\pi^2}{2ab}$ 26. $\frac{\pi}{6}$

7. DIFFERENTIAL EQUATION

** Homogeneous Differential Equation : $\frac{dy}{dx}=\frac{f_1(x,y)}{f_2(x,y)}$, where $f_1(x,y)$ & $f_2(x,y)$ be the homogeneous function of same degree.

** Linear Differential Equation :

(i) $\frac{dy}{dx}+py=q$, where p & q be the function of x or constant.

Solution of the equation is : $y.e^{\int p\,dx}=\int e^{\int p\,dx}.q\,dx$, where $e^{\int p\,dx}$ is Integrating Factor (I.F.)

(ii) $\frac{dx}{dy}+px=q$, where p & q be the function of y or constant.

Solution of the equation is: $x.e^{\int p\,dy}=\int e^{\int p\,dy}.q\,dy$, where $e^{\int p\,dy}$ is Integrating Factor (I.F.)

LONG-ANSWERTYPE QUESTIONS (4 or 6MARKS)

HOMOGENEOUS DIFFERENTIAL EQUATIONS

Solve the following differential equations

1. Solve $(x^2-y^2)dx+2xydy=0$, given that $y(1)=1$.

2. $\frac{dy}{dx}=\frac{x(2y-x)}{x(2y+x)}$, given that $y=1$, when $x=1$.

3. $(3xy-y^2)dx+(x^2+xy)dy=0$.

4. $\frac{dy}{dx}=\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)$, given that $y=0$, when $x=1$.

5. $x\frac{dy}{dx}=y-x\tan\left(\frac{y}{x}\right)$

6. $(x^3+y^3)dy-x^2ydx=0$

7. $x^2dy+(xy+y^2)dx=0$, given that $y=1$, when $x=1$.

8. $(x-y)\frac{dy}{dx}=x+2y$

9. $ydx+x\log\left(\frac{y}{x}\right)-2xdy=0$

10. $xdy-ydx=\sqrt{x^2+y^2}\,dx$

11. $2x^2\frac{dy}{dx}-2xy+y^2=0$

12. $x\frac{dy}{dx}-y+\sin\left(\frac{y}{x}\right)=0$, given that $y=\pi$, when $x=2$.

13. $2ye^{x/y}dx+\left(y-2xe^{x/y}\right)dy=0$, given that $y(0)=1$.

14. $x\frac{dy}{dx}.\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$, given that $y(1)=\frac{\pi}{2}$.

15. $\left(xe^{y/x}+y\right)dx=xdy$, given that $y=1$, when $x=1$.

ANSWERS

1. $x^2+y^2=2x$

2. $\log\left(\frac{2y^2-xy+x^2}{x^2}\right)+\frac{6}{\sqrt7}\tan^{-1}\left(\frac{4y-x}{\sqrt7\,x}\right)+2\log x=\log 2+\frac{6}{\sqrt7}\tan^{-1}\left(\frac{3}{\sqrt7}\right)$

3. $x\log\left(x^3y\right)=Cx$

4. $\cos\frac{y}{x}=\log x+1$

5. $x\sin\frac{y}{x}=C$

6. $-\frac{x^3}{3y^3}+\log|y|=C$

7. $3x^2y=y+2x$

8. $\log\left|x^2+xy+y^2\right|-2\sqrt3\tan^{-1}\left(\frac{2y+x}{\sqrt3 x}\right)=C$

9. $\left[\log\frac{y}{x}-1\right]=Cy$

10. $y+\sqrt{x^2+y^2}=Cx^2$

11. $\frac{2x}{y}=\log|x|+C$

12. $\left(\operatorname{cosec}\frac{y}{x}-\cot\frac{y}{x}\right)=2$

$13.\ 2e^{x/y} + \log|y| = C$ $\quad$ $14.\ \cos\frac{y}{x} = \log|x|$ $\quad$ $15.\ e^{\frac{y}{x}}.\log|x| - e^{\frac{y}{x}-1} + 1 = 0$

LINEAR DIFFERENTIAL EQUATIONS

Solve the following differential equations :

1. $\cos^2 x\frac{dy}{dx} + y = \tan x$
2. $x\frac{dy}{dx} + y = x\log x;\ x \neq 0$
3. $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$
4. $x\log x\frac{dy}{dx} + y = 2\log x$
5. $\frac{dy}{dx} + y = \cos x - \sin x$
6. Solve : $\frac{dy}{dx} + y\cot x = 4x\,\text{cosec}\,x,\ (x \neq 0)$, given that $y\left(\frac{\pi}{2}\right) = 0$.
7. $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$
8. $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$
9. $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$
10. $(3x^2+y)\frac{dx}{dy} = x,\ x > 0$, when $x = 1,\ y = 1$.
11. $xdy + (y - x^3)dx = 0$
12. $\frac{dy}{dx} + 2y\tan x = \sin x;\ y\left(\frac{\pi}{3}\right) = 0$
13. $(1+x^2)dy + 2xydx = \cot x dx\ ;\ x \neq 0$
14. $\frac{dx}{dy} + x\cot y = 2y + y^2\cot y,\ (y \neq 0)$, given that $y\left(\frac{\pi}{3}\right) = 0$.
15. $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

ANSWERS

1. $ye^{\tan x} = e^{\tan x}(\tan x - 1) + C$
2. $y = \frac{x}{2}\left(\log x - \frac{1}{2}\right) + C$
3. $y = (\tan^{-1} x - 1) + Ce^{-\tan^{-1} x}$
4. $y\log x = (\log x)^2 + C$
5. $y = \cos x + Ce^{-x}$
6. $y\sin x = 2x^2 - \frac{\pi^2}{2}$
7. $y = \frac{1}{x^2-1}\log\left|\frac{x-1}{x+1}\right| + \frac{C}{x^2-1}$
8. $y\log x = -\frac{2}{x}[\log x + 1] + C$
9. $(x^2+1)y = \frac{x}{2}\sqrt{x^2+4} + 2\log\left|x + \sqrt{x^2+4}\right| + C$
10. $y = 3x^2 - 2x$
11. $xy = \frac{x^4}{4} + C$
12. $y = \cos x - 2\cos^2 x$
13. $y = \frac{\log\sin x}{1+x^2} + \frac{C}{1+x^2}$
14. $x\sin y = y^2\sin y - \frac{\pi^2}{4}$
15. $y = \frac{e^{\tan^{-1} x}}{2} + Ce^{-\tan^{-1} x}$

8. VECTOR ALGEBRA

* Position vector of point $A(x,y,z) = \overrightarrow{OA} = x\hat{i} + y\hat{j} + z\hat{k}$
* If $A(x_1, y_1, z_1)$ and point $B(x_2, y_2, z_2)$ then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
* If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$; $|\vec{a}| = \sqrt{x^2+y^2+z^2}$
* Unit vector parallel to $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

* Scalar Product (dot product) between two vectors : $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$; θ is angle between the vectors

* $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

* If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then $\vec{a}\cdot\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$

* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a}\cdot\vec{b} = 0$

* $\vec{a}\cdot\vec{a} = |\vec{a}|^2$

* Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

* Vector product between two vectors : $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$; $\hat{n}$ is the normal unit vector which is perpendicular to both $\vec{a}$ & $\vec{b}$

* $\hat{n} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a}\times\vec{b} = 0$

* Area of triangle (whose sides are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}|\vec{a}\times\vec{b}|$

* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$) $= |\vec{a}\times\vec{b}|$

* Area of parallelogram (whose diagonals are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}|\vec{a}\times\vec{b}|$

* Scalar triple product of vectors $\vec{a}$, $\vec{b}$ and $\vec{c} = [\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

* Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplaner iff $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

LONG-ANSWERTYPE QUESTIONS (4MARKS)

1. Find a vector of magnitude 5 units, perpendicular to each of the vectors($\vec{a} + \vec{b}$) ,($\vec{a} - \vec{b}$) where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

2. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

3. The dot products of a vector with the vectors $\hat{i} - 3\hat{j}$, $\hat{i} - 2\hat{j}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0 , 5 and 8 respectively. Find the vectors.

4. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d} = 18$.

5. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

6. If a unit vector $\vec{a}$ makes angles $\pi/4$ with $\hat{i}$, $\pi/3$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the component of $\vec{a}$ and angle θ.

7. If with reference to the right handed system of mutually perpendicular unitvectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, $\vec{\alpha} = 3\hat{i}+4\hat{j}+5\hat{k}$, $\vec{\beta} = 2\hat{i}+\hat{j}-4\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

8. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

9. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$.

10. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}-3\hat{j}-5\hat{k}$, respectively form the vertices of a right angledtriangle.

11. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = 0$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

12. Three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c} = 0$. Evaluate $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ if $|\vec{a}| = 1$, $|\vec{b}| = 4$, $|\vec{c}| = 2$.

13. Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.

14. Show that the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar if and only if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

ANSWERS

1. $-\frac{5}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$ **3.** $\hat{i}+2\hat{j}+\hat{k}$ **4.** $64\hat{i}-2\hat{j}-28\hat{k}$ **6.** $\frac{1}{\sqrt{2}}\vec{a}$, $\frac{1}{2}\vec{b}$, $\frac{1}{2}\vec{c}$ $\theta = \frac{\pi}{3}$.

7. $2\hat{i}+\hat{j}-4\hat{k} = \left(-\frac{3}{5}\hat{i}-\frac{4}{5}\hat{j}-\hat{k}\right)+\left(\frac{13}{5}\hat{i}+\frac{9}{5}\hat{j}-3\hat{k}\right)$ **8.** $\lambda = 1$. **9.** $5\sqrt{2}$.

11. $-\frac{3}{2}$ **12.** $-\frac{21}{2}$

9. THREE DIMENSIONAL GEOMETRY

** Direction cosines and directionratios:

If a line makes angles α, β and γ with x, y and z axes respectively the $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the directioncosines denoted by l, m, n respectively and $l^2+m^2+n^2=1$

**Any three numbers proportion al to direction cos ines are direction ratios denoted by a, b, c

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c} \qquad l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}},$$

* Direction ratios of a line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ may be taken as $x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$
* Angle between two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 is given by

$$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2 = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{\left(a_1^2 + b_1^2 + c_1^2\right)\left(a_2^2 + b_2^2 + c_2^2\right)}}$$

* For parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and for perpendicular lines $a_1a_2 + b_1b_2 + c_1c_2 = 0$ or $l_1l_2 + m_1m_2 + n_1n_2 = 0$

** STRAIGHT LINE :

* Equation of line passing through a point (x_1, y_1, z_1) with direction cosines a, b, c: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$
* Equation of line passing through a point (x_1, y_1, z_1) and parallel to the line: $\frac{x - \alpha}{a} = \frac{y - \beta}{b} = \frac{z - \gamma}{c}$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

* Equation of line passing through two point (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$
* Equation of line (Vector form)

Equation of line passing through a point $\vec{a}$ and in the direction of $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$

* Equation of line passing through two points $\vec{a}$ & $\vec{b}$ and in the direction of $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$
* Shortest distance between two skew lines: if lines are $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$

then Shortest distance $= \frac{\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}$; $\vec{b_1} \times \vec{b_2} \neq 0$

$$\frac{\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b_1}\right|}{\left|\vec{b_1}\right|} \quad ; \vec{b_1} \times \vec{b_2} = 0$$

** PLANE :

* Equation of plane is $ax + by + cz + d = 0$ where a, b & c are direction ratios of normal to the plane
* Equation of plane passing through a point (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
* Equation of plane in intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a, b, c are intercepts on the axes
* Equation of plane in normal form $lx + my + nz = p$ where l, m, n are direction cosines of normal to the plane p is length of perpendicular form origin to the plane

LONG-ANSWERTYPE QUESTIONS (4 OR 6 MARKS)

1. Find the equation of the line passing through (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 2, 1).

2. Find the value of λ so that the lines $\frac{1 - x}{3} = \frac{7y - 14}{2\lambda} = \frac{5z - 10}{11}$ and $\frac{7 - 7x}{3\lambda} = \frac{y - 5}{1} = \frac{6 - z}{5}$.
are perpendicular to each other.

3. Find the shortest distance between the following lines : $\frac{x - 3}{1} = \frac{y - 5}{-2} = \frac{z - 7}{1}$ and $\frac{x + 1}{7} = \frac{y + 1}{-6} = \frac{z + 1}{1}$

4. Find the shortest distance between the following two lines:
$\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k}$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

5. Show that the lines $\frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4}$ and $\frac{x - 4}{5} = \frac{y - 1}{2} = z$ intersect. Find their point of intersection.

6. Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).

7. Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

8. Show that the four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .

9. Find the equation of the plane passing through the point (– 1, – 1, 2) and perpendicular to each of the following planes: 2x + 3y – 3z = 2 and 5x – 4y + z = 6.

10. Find the equation of the plane passing through the points (2, 1, –1) and (–1, 3, 4) and perpendicular to the plane x – 2y + 4z = 10.

11. Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, – 1, 2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

12. Find the equation of the plane passing through the intersection of the planes 2x – 3y + z + 4 = 0 and x – y + z + 1 = 0 and perpendicular to the plane x +2y – 3z +6 = 0.

13. Find the equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}+\hat{j}+3\hat{k})=7$ and $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and passing through the point (2, 1, 3).

14. Find the equation of the plane passing through the intersection of the planes 2x + y –z = 3 and 5x –3y +4z + 9 = 0 and is parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

15. Find the equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at unit distance from origin.

16. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1, 3, 4) from the plane 2x – y + z + 3 = 0. Find also, the image of the point in the plane.

ANSWERS

1. $\frac{x-1}{-2}=\frac{y+1}{-1}=\frac{z-1}{1}$ **2.** $\lambda=7$ **3.** $2\sqrt{29}$ **4.** $\frac{3\sqrt{2}}{2}$ units

5. $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$ **6.** $(-2\ ,\ -1,\ 3)$ or $\left(\frac{56}{17},\frac{43}{17},\frac{77}{17}\right)$ **7.**(1, 0, 7)

8.5x – 7y + 11z +4 = 0 **9.** $9x+17y+23z=20$ **10.**18x + 17y + 4z –49 = 0

11.x – 19y – 11z = 0**.** **12.**x – 5y – 3z – 23 = 0. **13.** $\vec{r}.(2\hat{i}-13\hat{j}+3\hat{k})=0$

14.7x +9y – 10z – 27 = 0 **15.** $\vec{r}.(2\hat{i}+\hat{j}+2\hat{k})+3=0$ and $\vec{r}.(-\hat{i}+2\hat{j}-2\hat{k})+3=0$.

16. $(1,3,0)$ and $(-1,4,-1)$

10. LINEAR PROGRAMMING PROBLEM

** Solving linear programming problem using **Corner Point Method**. The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.

2. Evaluate the objective function Z = *ax* + *by* at each corner point. Let M and *m*, respectively denote the largest and smallest values of these points.

3. (i) When the feasible region is **bounded**, M and *m* are the maximum and minimum values of Z.

(ii) In case, the feasible region is **unbounded**, we have:

4. (a) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

(b) Similarly, *m* is the minimum value of Z, if the open half plane determined by ax + by <mhas no point in common with the feasible region. Otherwise, Z has no minimum value

LONG-ANSWERTYPE QUESTIONS (6 MARKS)

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

2. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

3. A toy company manufactures two types of dolls, A and B. Market tests and availableresources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

4. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

5. There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

6. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

7.Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

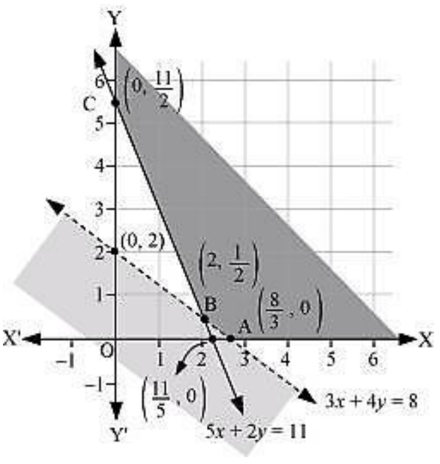
Transportation cost per quintal (in Rs)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

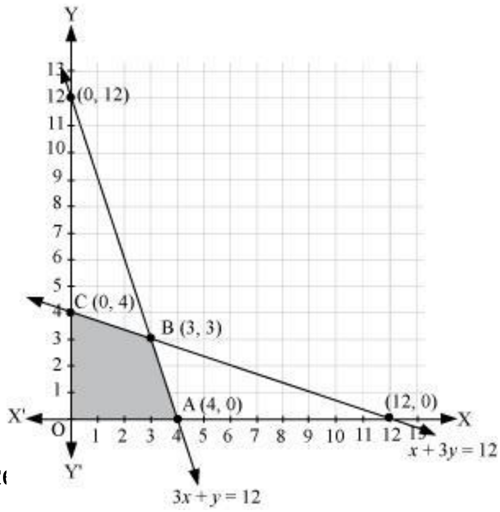
ANSWERS

1. LPP is to minimise Z = 60x + 80ysubject to the constraints, $3x + 4y \geq 8$,$5x + 2y \geq 11$, $x, y \geq 0$

<table>
<tr><th>Corner point</th><th>Z = 60x + 80y</th><th></th></tr>
<tr><td>$A\left(\frac{8}{3}, 0\right)$</td><td>160</td><td rowspan="2">Minimum</td></tr>
<tr><td>$B\left(2, \frac{1}{2}\right)$</td><td>160</td></tr>
<tr><td>$C\left(0, \frac{11}{2}\right)$</td><td>440</td><td></td></tr>
</table>



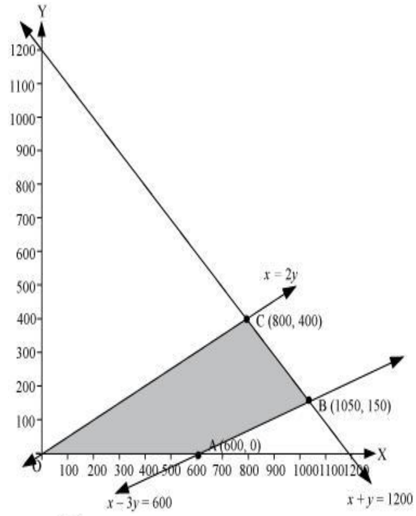
2. L.P. P is to maximise Z = 17.5x + 7ysubject to the constraints, x + 3y ≤12 , 3x + y ≤ 12, $x, y \geq 0$



Corner point	Z = 17.5x + 7y	
O(0, 0)	0	
A(4, 0)	70	
B(3, 3)	73.5	→ Maximum
C(0, 4)	28	

Thus, 3 packages of nuts and 3 packages of bolts should be produc each day to get the maximum profit of Rs 73.50.

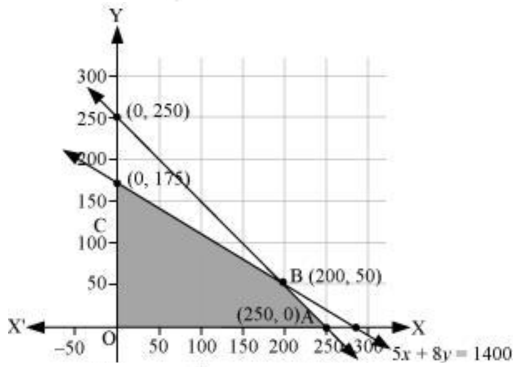
3. LPP is to maximize z = 12x + 16y subject to the constraints, x + y ≤ 1200, $y \leq \frac{x}{2}$, x – 3y ≤ 600, $x, y \geq 0$



Corner point	z = 12x + 16y	
A (600, 0)	7200	
B (1050, 150)	15000	
C (800, 400)	16000	→ Maximum

∴800 and 400 dolls of type A and type B should be produced respectively to get the maximum profit of Rs 16000.

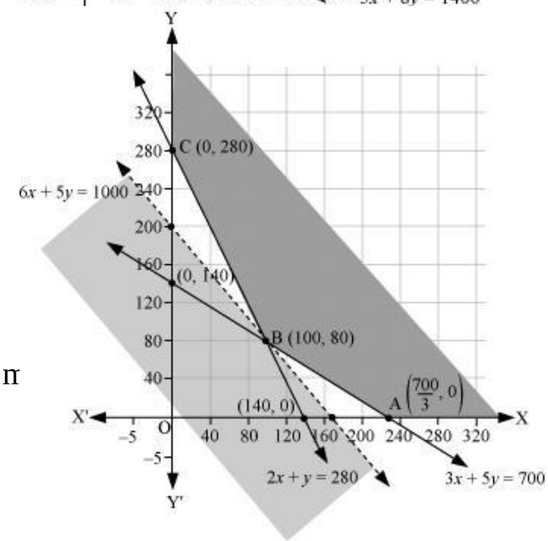
4.LPP is to maximize Z = 4500x + 5000y subject to the constraints, 5x + 8y ≤ 1400, x + y ≤ 250, $x, y \geq 0$



Corner point	Z = 4500x + 5000y	
A(250, 0)	1125000	
B(200, 50)	1150000	→ Maximum
C(0, 175)	875000	

∴the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

5. LPP is to minimize Z = 6x + 5y subject to the constraints, 2x + y ≥280 ,3x + 5y≥700 , $x, y \geq 0$



Corner point	Z = 6x + 5y	
$A\left(\frac{700}{3}, 0\right)$	1400	
B(100, 80)	1000	→ Minimum
C(0, 280)	1400	

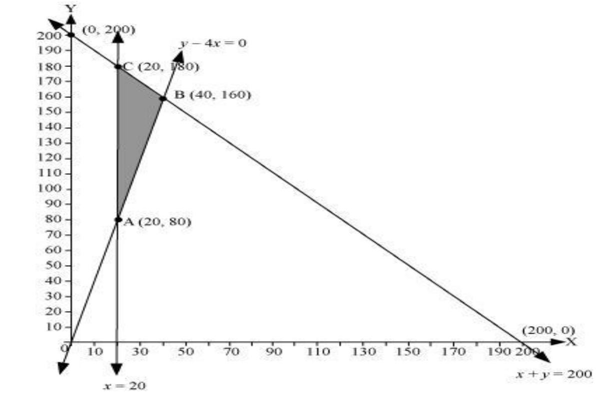
∴100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to m is Rs 1000.

6. LPP is to maximize z = 1000x + 600y subject to the constraints, x + y ≤ 200, x ≥ 20, y ≥ 4x, $x, y \geq 0$

Corner point	z = 1000x + 600y	
A (20, 80)	68000	

B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

∴40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

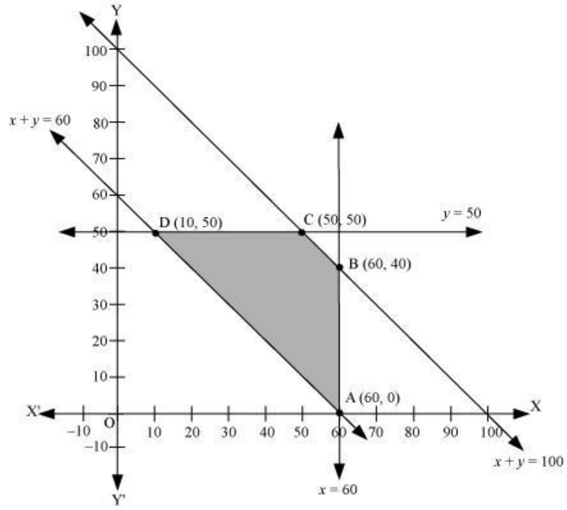


7. LPP is to minimize z = 2.5x + 1.5y + 410 subject to the constraints,

$x + y \le 100,\ x \le 60,\ y \le 50,\ x + y \ge 60,\ x, y \ge 0$

Corner point	z = 2.5x + 1.5y + 410	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	→ Minimum

∴the amount of grain transported from A to D, E, and F is 10 quintals, 50 quintals, and 40 quintals respectively and from B to D, E, and F is 50 quintals, 0 quintals, and 0 quintals respectively. The minimum cost is Rs 510.



11. PROBABILTY(BAYES THEOREM)

**If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$P(E|F) = \dfrac{P(E \cap F)}{P(F)}$ provided $P(F) \neq 0$

** Multiplication rule of probability : $P(E \cap F) = P(E)\,P(F|E)$

$= P(F)\,P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.

** Independent Events : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.

Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E)\,.\,P(F)$.

** Bayes' Theorem : If $E_1, E_2, ..., E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, ..., E_n$ are pair wise disjoint and $E_1 \cup E_2 \cup ... \cup E_n = S$ andA is any event of nonzero probability, then

$P(Ei|A) = \dfrac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j).P(A|E_j)}$ for any i = 1, 2, 3, ..., n

** The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	...	x_n
P(X) :	p_1	p_2	...	p_n

where, $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$, i = 1, 1, 2,...,

** **Binomial distribution:** The probability of x successes P (X = x) is also denoted by P (x) and is given by $P(x) = {}^{n}C_{x}\, q^{n-x} p^{x}$, x = 0, 1,..., n. (q = 1 – p)

1. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

2. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

3. Suppose that the reliability of a HIV test is specified as follows:

Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV –ive but 1% are diagnosed as showing HIV +ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ive. What is the probability that the person actually has HIV?

4. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively **3/10, 1/5, 1/10** and **2/5**. The probabilities that he will be late are **1/4**,**1/3** and **1/12**, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

5.In answering a question on a multiple choice test, a student either knows the answer or guesses. Let **3/4** be the probability that he knows the answer and**1/4** be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability **1/4**. What is the probability that the student knows the answer given that he answered it correctly?

6.A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive ?

7.An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

8.Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

9.A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

10. Probability that A speaks truth is **4/5**. A coin is tossed. A reports that a head appears. Find probability that actually there was head .

11.Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

ANSWERS

1. $\frac{2}{3}$ **2.** $\frac{2}{3}$ **3.** $\frac{90}{1089} = 0.083$ **4.** $\frac{1}{2}$ **5.** $\frac{12}{13}$ **6.** $\frac{22}{133}$

7. $\frac{1}{52}$ **8.** $\frac{16}{31}$ **9.** $\frac{11}{50}$ **10.** $\frac{4}{5}$ **11.** $\frac{8}{11}$

For any suggestion , correction or modification please email to : rajiv.ranjan1965@gmail.com